Fill in the following identities. **SCORE: _____ / 14 PTS**

[a] SUM OF ANGLES IDENTITY:

$\sin(x+y) =$ $\sin x \cos y + \cos x \sin y$

[b] DIFFERENCE OF ANGLES IDENTITY:

$\tan(x-y) =$ $\dfrac{\tan x - \tan y}{1+\tan x \tan y}$

[c] POWER REDUCING IDENTITY:

$\cos^2 x =$ $\frac{1}{2}(1+\cos 2x)$

[d] HALF ANGLE IDENTITY:

$\sin\frac{1}{2}x =$ $\pm\sqrt{\dfrac{1-\cos x}{2}}$

[e] NEGATIVE ANGLE IDENTITY:

$\sec(-x) =$ $\sec x$

[f] PYTHAGOREAN IDENTITY:

$\tan^2 x =$ $\sec^2 x - 1$

[g] DOUBLE ANGLE IDENTITY:
WRITE ALL 3 VERSIONS

$\cos 2x =$ $\cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

If $\cos t = -\frac{4}{5}$ and $\pi < t < \frac{3\pi}{2}$, find the values of the following expressions. **SCORE: _____ / 30 PTS**

Write each final answer as a single fraction in simplest form, including rationalizing the denominator.

[a] $\tan\frac{1}{2}t = \dfrac{\sin t}{1+\cos t}$

$= \dfrac{-\frac{3}{5}}{1-\frac{4}{5}}$

$= \dfrac{-\frac{3}{5}}{\frac{1}{5}}$

$= -3$

[b] $\sin 2t = 2\sin t\cos t$

$= 2(-\frac{3}{5})(-\frac{4}{5})$

$= \frac{24}{25}$

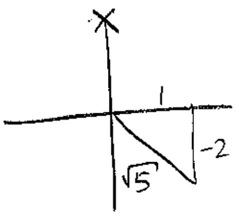
[c] $\cos(\underbrace{\arctan(-2)}_{}-t) = \cos(x-t)$

$x = \arctan(-2)$

$= \cos x\cos t + \sin x\sin t$

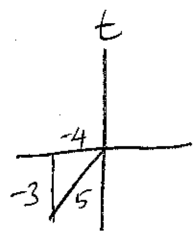
$\tan x = -2$ AND $x \in Q_4$

$= \frac{1}{\sqrt{5}}\cdot\frac{-4}{5} + \frac{-2}{\sqrt{5}}\cdot\frac{-3}{5}$




$= \dfrac{2}{5\sqrt{5}}$

$= \dfrac{2\sqrt{5}}{25}$



Solve the equation $7-4\cos 3x = 6(1-\cos 3x)$.

**SCORE: _____ / 14 PTS**

$$7-4\cos 3x = 6-6\cos 3x$$

$$2\cos 3x = -1$$

$$\cos 3x = -\frac{1}{2}$$

$$3x = \frac{2\pi}{3} + 2n\pi, \quad \frac{4\pi}{3} + 2n\pi$$

$$x = \frac{2\pi}{9} + \frac{2n\pi}{3}, \quad \frac{4\pi}{9} + \frac{2n\pi}{3}$$

Prove the identity $\dfrac{(\csc B-\cot B)^2+1}{\sec B\csc B-\cot B\sec B}=2\cot B$.

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$$= \frac{\csc^2 B - 2\csc B\cot B + \cot^2 B + 1}{\sec B\,(\csc B - \cot B)}$$

$$= \frac{2\csc^2 B - 2\csc B\cot B}{\sec B\,(\csc B - \cot B)}$$

$$= \frac{2\csc B\,\cancel{(\csc B - \cot B)}}{\sec B\,\cancel{(\csc B - \cot B)}}$$

$$= 2\cdot\frac{1}{\sin B}\cdot\cos B = 2\cot B$$

Rewrite $\sin^4 x$ using only the first powers of cosine (and constants and the 4 basic arithmetic operations). **SCORE: _____ / 14 PTS**

Simplify your final answer, which must **NOT** be in factored form, and must **NOT** involve any other trigonometric functions.

$$\sin^4 x = \left(\tfrac{1}{2}(1-\cos 2x)\right)^2$$

$$= \tfrac{1}{4}(1-2\cos 2x+\cos^2 2x)$$

$$= \tfrac{1}{4}\left(1-2\cos 2x+\tfrac{1}{2}(1+\cos 4x)\right)$$

$$= \tfrac{1}{8}(3-4\cos 2x+\cos 4x)$$

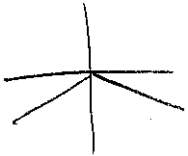
Solve the equation $2\cos 2x + 7\sin x = 0$ algebraically. **SCORE: _____ / 14 PTS**

$$2(1-2\sin^2 x) + 7\sin x = 0$$

$$2 + 7\sin x - 4\sin^2 x = 0$$

$$(2-\sin x)(1+4\sin x) = 0$$

$$\sin x = \cancel{2} \text{ OR } \sin x = -\tfrac{1}{4} \longrightarrow x \in Q_3, Q_4$$



$$x = \pi + \sin^{-1}\tfrac{1}{4} + 2n\pi$$

$$\text{OR } 2\pi - \sin^{-1}\tfrac{1}{4} + 2n\pi$$

$$= 3.3943 + 2n\pi \text{ OR } 6.0305 + 2n\pi$$